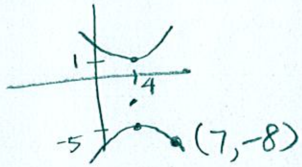
Consider the hyperbola which passes through the point $(7, -8)$, with vertices $(4, -5)$ and $(4, 1)$. **SCORE: _____ / 8 PTS**

[a] Find the standard form of the equation of the hyperbola.



$$\text{CENTER} = \left(4, \frac{-5+1}{2}\right) = (4, -2) \quad \tfrac{1}{2}$$

$$a = \frac{1 - -5}{2} \text{ OR } 1 - -2 = 3 \quad 1$$

$$\frac{(y+2)^2}{9} - \frac{(x-4)^2}{b^2} = 1$$

$$\frac{(-8+2)^2}{9} - \frac{(7-4)^2}{b^2} = 1$$

$$4 - \frac{9}{b^2} = 1 \quad 1$$

$$\frac{9}{b^2} = 3$$

$$b^2 = 3$$

$$\frac{(y+2)^2}{9} - \frac{(x-4)^2}{3} = 1$$

$1 \quad 1\tfrac{1}{2} \quad \tfrac{1}{2}$

[b] Find the **slope-point form** of the equation of the asymptotes.

$$\text{SLOPE} = \pm\frac{\sqrt{9}}{\sqrt{3}} = \pm\sqrt{3}$$

$$y+2 = \pm\sqrt{3}(x-4)$$

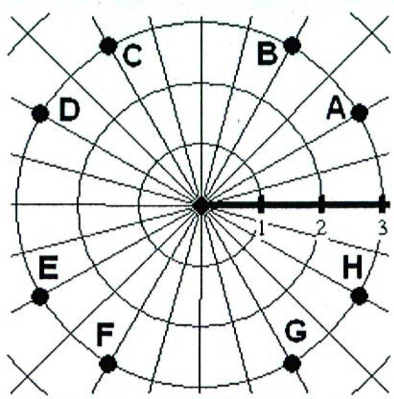
$\tfrac{1}{2} \quad \tfrac{1}{2} \quad 1 \quad \tfrac{1}{2}$

Fill in the blanks. **SCORE: _____ / 3 PTS**

[a] The shape of the graph of $9x^2 + 6x - 5y^2 + 8y - 10 = 0$ is a/an HYPERBOLA. $1\tfrac{1}{2}$

[b] The shape of the graph of $7x^2 + 7x - 11y + 13 = 0$ is a/an PARABOLA. $1\tfrac{1}{2}$

Fill in the blanks using the graph on the right. **SCORE: _____ / 4 PTS**



[a] The polar co-ordinates $(-3, \frac{5\pi}{6})$ refers to point H. 1

[b] The polar co-ordinates $(3, -\frac{4\pi}{3})$ refers to point C. 1

[c] The polar co-ordinates $(-3, \frac{2\pi}{3}$ OR $\frac{-4\pi}{3})$ refers to point G. 2

Convert the polar equation $r^2 = \sin 2\theta$ into rectangular form. **SCORE: _____ / 6 PTS**

$r^2 = \underline{2\sin\theta\cos\theta}$ (2)

$\underline{r^2 \cdot r^2 = 2(r\sin\theta)(r\cos\theta)}$ (2)

$\underbrace{(x^2+y^2)^2}_{(1)} = \underbrace{2xy}_{(1)}$

Convert the rectangular coordinates $(2\sqrt{3}, -6)$ into polar coordinates. **SCORE: _____ / 3 PTS**

Write your final answer in proper notation.

$r^2 = 12 + 36 = 48 \rightarrow r = \underline{4\sqrt{3}}$ (1)

$\cos\theta = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{2}$

$\sin\theta = \frac{-6}{4\sqrt{3}} = -\frac{\sqrt{3}}{2}$

$\theta = -\frac{\pi}{3}$ OR $\frac{5\pi}{3}$

OR (1 1/2)

$\underline{\left(4\sqrt{3}, -\frac{\pi}{3}\right)}$ OR $\left(4\sqrt{3}, \frac{5\pi}{3}\right)$

(1/2) FOR PROPER FORMAT [r FIRST, θ SECOND, PARENTHESES + COMMA]

Test the polar equation $r = 1 - 2\sin\theta$ for symmetry with respect to the polar axis. **SCORE: _____ / 6 PTS**

State your conclusion clearly.

$(r, -\theta)$

$\underline{r = 1 - 2\sin(-\theta)}$ (1)

$\underline{r = 1 + 2\sin\theta}$ (1)

NO CONCLUSION

$(-r, \pi - \theta)$

$\underline{-r = 1 - 2\sin(\pi - \theta)}$ (1)

$-r = 1 - 2[\sin\pi\cos\theta - \cos\pi\sin\theta]$

$-r = 1 - 2\sin\theta$

$\underline{r = -1 + 2\sin\theta}$ (2)

NO CONCLUSION. (1)

Convert the rectangular equation $2x - 3y + 4 = 0$ into polar form. **SCORE: _____ / 5 PTS**

Remember to solve for *r*.

$\underline{2r\cos\theta - 3r\sin\theta + 4 = 0}$ (2)

$\underline{r(2\cos\theta - 3\sin\theta) = -4}$ (2)

$\underbrace{r = \frac{-4}{2\cos\theta - 3\sin\theta}}_{(1)}$ OR $\frac{4}{3\sin\theta - 2\cos\theta}$